

A assoc alg $HH^*(A; A) = HH^*(A)$

e.g. $HH^0(A) = \{z \in A : az = za\} = Z(A)$

In gen, one could def the center $Z(M)$ for \forall monoid M as $ZM = \{z \in M = mz : \forall m \in M\}$

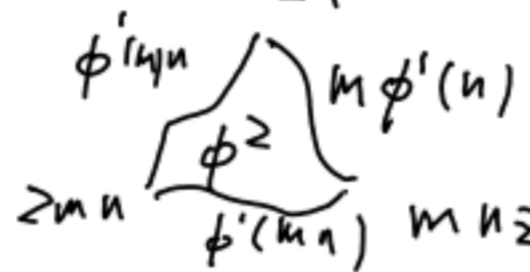
or. for category \mathcal{C} , $Z(\mathcal{C}) = \text{Nat}(Id_{\mathcal{C}}, Id_{\mathcal{C}})$

Q. what if we show in topology, or more sensible def of $Z(\mathcal{C})$

Homotopy coherent center

M top monoid. consider:

$Z = \phi^0$, $\forall m \in M : \phi^1(\text{point } m) : \Delta^1 \rightarrow M$ $zm \sim mz$

$\forall m, n$ $\phi^2(m, n)$  ... compat

For category, \mathcal{C} : top cat ($\text{Mor}(X, Y)$ is top sp)
 $\pi^k \mathcal{C} = \pi \text{Map}(\mathcal{C}(X_1, X_0), X_1 \dots X_k, \mathcal{C}(X_k, X_{k-1}), \mathcal{C}(X_k, X_0))$

$\leadsto \pi^* \mathcal{C}$ cosimplicial sp.

$\text{Tot } \pi^* \mathcal{C} = \text{Hom}(\Delta^0, \pi^* \mathcal{C}) =: Z^{\text{hp}}(\mathcal{C})$

Thm $Z^{\text{hp}}(\mathcal{C})$ is a homotopy comm Assoc monoid

Thm Equiv cats have 'equiv centers'

Example when $\mathcal{C} = \text{top grp } G$
 $Z(G) = \text{fix ptr for } \text{Ad}(G)$
 $= \text{Map}_G^*(G, \text{Ad}G)$

$Z^{top}(G) \simeq \text{Map}_G(\mathbb{E}G, (G; \text{Ad}G))$: homotopy fixed pt
 from $\mathbb{E}G \times_{\text{Ad}G} \sim \wedge BG$
 \downarrow \downarrow
 BG BG

$\leadsto Z^{top} \simeq \Omega(\text{Map}(BG, BG), id)$

Thm (Dwyer + Wilkerson) G conn lie grp

$\leadsto BZG \rightarrow BZ^{top}G$ is isom in $H_*(\sim; \mathbb{F})$

Example $A : \{\text{plx line blls over } S^1\} = \text{Map}(S^1; \mathbb{C}P^\infty)$
 $\leadsto ZA = A \rightarrow Z^{top}(A)$ is not equivalence.

$\pi_0(A) \rightarrow \pi_1(\text{Map}(BA, BA)) = \mathbb{Z}$

Comparison of $\pi_0 Z^{top} \mathcal{C} \rightarrow \mathbb{Z} H_0 \mathcal{C}$.

\exists spec seq $E^{s,t} \Rightarrow \pi_{t-s}(Z^{top} \mathcal{C})$ w/ $E_2^{0,0} = \mathbb{Z}(H_0 \mathcal{C})$

but w/ nontriv obstruction

Example X sp, $\mathcal{C} = GX$ loop groupoid.

$\leadsto H_0 \mathcal{C} = \pi_1 X$ fundamental groupoid

$\mathbb{Z} H_0 \mathcal{C} = \mathbb{Z}(\pi_1 X)$ (supposing X is conn)

$\pi_0 Z^{top} \mathcal{C} = \pi_1 \text{Map}(X, X)$

Not same for $X = \mathbb{R}P^2$ ($0 \rightarrow \mathbb{Z}/2$)

Mo' examples

$$X = K(A, n) \rightsquigarrow Z^{\text{top}}(GX) = K(A, n-1)$$

can cook up arbitrary high $\pi_x(Z^{\text{top}}e)$

$$X = S^n \quad n \geq 2 \rightsquigarrow Z(H_0 GX) = 0$$

$$\pi_0(Z^{\text{top}} GX) = \mathbb{Z}/2$$