Szymik CPH seminar

A assoc alg $\quad HH^*(A;A) = HH^*(A)$

e.g. $\quad HH^0(A) = \{z \in A : az = za\} = Z(A)$

In gen, one could def the center $Z(M)$ for

$\forall$ monoid M as $ZM = \{zm = mz : \forall m \in M\}$

or. for category $\mathcal{C}$, $Z(\mathcal{C}) = \mathrm{Nat}(\mathrm{Id}_{\mathcal{C}}, \mathrm{Id}_{\mathcal{C}})$

Q. what if we slow in topology, or more sensible def of $Z(\mathcal{C})$

Homotopy coherent center

M top monoid. consider:

$z = \phi^0$, $\quad \forall m \in M : \underset{mzn}{\phi^1(m)} : \Delta^1 \to M \quad zm \rightsquigarrow mz$

$\forall m, n \quad \phi^2(m,n)$

$$\begin{tikzcd} & mzn \arrow[dr, "m\phi^1(n)"] & \\ zmn \arrow[ur, "\phi^1(m)n"] \arrow[rr, "\phi^1(mn)"'] \arrow[rr, phantom, bend left=20, "\phi^2"] & & mnz \end{tikzcd}$$

$\cdots$ compat

For category, $\mathcal{C}$: top cat ($\mathrm{Mor}(X,Y)$ is top sp)

$\Pi^k\mathcal{C} = \prod \mathrm{Map}(\mathcal{C}(X_1, X_0) \times \cdots \times \mathcal{C}(X_k, X_{k-1}), \mathcal{C}(X_k, X_0))$

$\rightsquigarrow \Pi^*\mathcal{C}$ cosimplicial sp.

$\mathrm{Tot}\,\Pi^\bullet\mathcal{C} = \mathrm{Hom}(\Delta^\bullet, \Pi^\bullet\mathcal{C}) =: Z^{top}(\mathcal{C})$

Thm $\quad Z^t(\mathcal{C})$ is a homotopy comm A_∞ monoid

Th'm Equiv cats have 'equiv centers'

Szymik no. 2.

Example when $\mathcal{C}$: top grp G

$Z(G) =$ fix pts for $Ad(G)$

: $Map_G(*, (G, AdG))$

$\downarrow$

$Z^{top}(G) \simeq Map_G(EG, (G; AdG))$: homotopy fixed pt

from $EG \times_G AdG \sim \Lambda BG$

$\downarrow BG \qquad \downarrow BG$

$\rightsquigarrow Z^{top} \simeq \Omega(Map(BG, BG), id)$

Th'm (Dwyer + Wilkerson) G conn lie grp

$\rightsquigarrow BZG \to BZ^{top}G$ is isom in $H_*(-; \mathbb{F}_p)$

Example A : {cplx line bdls over S^1} $= Map(S^1; \mathbb{CP}^\infty)$

$\rightsquigarrow ZA = A \to Z^{top}(A)$ is not equivalence.

$\pi_0(A) \to \pi_1(Map(BA, BA)) = \mathbb{Z}$

- comparison of $\pi_0 Z^{top}\mathcal{C} \to Z H_0 \mathcal{C}$.

$\exists$ spec seq $E_2^{s,t} \Rightarrow \pi_{t-s}(Z^{top}\mathcal{C})$ w/ $E_2^{0,0} = Z(H_0\mathcal{C})$

but w/ nontriv obstruction

Example X sp, $\mathcal{C} = GX$ loop groupoid.

$\rightsquigarrow H_0\mathcal{C} = \Pi_1 X$ fundamental groupoid

$Z H_0 \mathcal{C} = Z(\pi_1 X)$ (supposing X is conn)

$\pi_0 Z^{top}\mathcal{C} = \pi_1 Map(X, X)$

not same for $X = \mathbb{RP}^2$ $(0 \to \mathbb{Z}/2)$

Szymik no. 3

Mo' examples

$X = K(A, n) \rightsquigarrow Z^{top}(GX) = K(A, n-1)$

can cook up arbitrary high $\pi_*(Z^{top} e)$

$X = S^n \quad n \geqslant 2 \rightsquigarrow Z(H_0\, GX) = 0$

$\pi_0(Z^{top}\, GX) = \mathbb{Z}/2$