Wahl HH seminar no. 1

CHF (Kontsevich-Soibelman)

A (dg-) algebra ⇝ $(C_*(A,A), b)$, $(C^*(A,A), \delta)$

$D \in C^*(A,A)$ ⇝ ops: $L_D, \iota_D, \cap D$ on $C_*(A,A)$

The CHF: $[b - B, \iota_D] = L_D - \iota_{\delta(D)}$

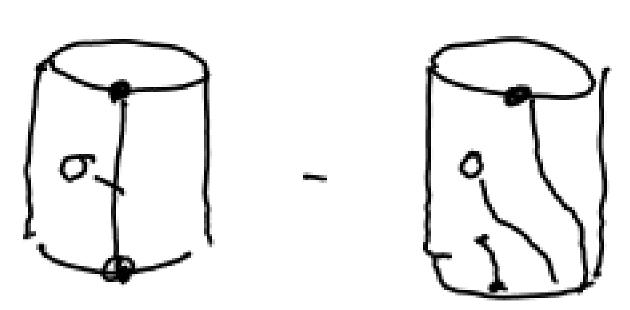
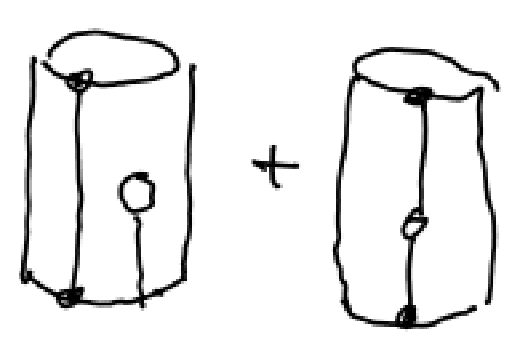
i.e. in the hom chain cplx

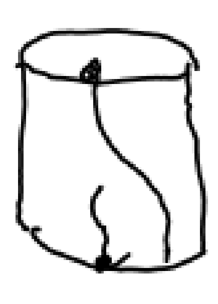
$$\mathcal{O}(1,1) = Hom(C_*(A,A), C_*(A,A)) = \left(\prod_{j \geq 1} Hom(A^{\otimes j}, \bigoplus_{k \geq 1} A^{\otimes k}), d\right)$$

d is s.t. $\mathcal{O}(1,1) \otimes C_*(A,A) \to C_*(A,A)$ is a chain map ($df = [b, f]$)

A subcplx of ops

Claim: $\exists$ chains of "punctured cylinders" Cyl_*, chain map $Cyl_* \otimes C^*(A,A) \to \mathcal{O}(1,1)$ s.t. $L_D, \iota_D, \ldots$ are in the img

<u>Example</u> "ι_D" ←→  , "L_D" ←→ 

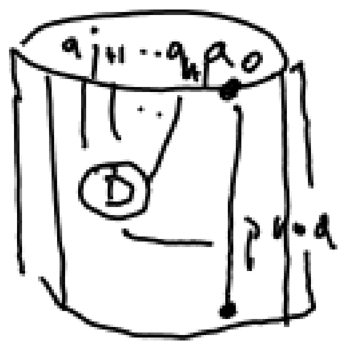
"B" ←→ 

Boundary on Cyl = Σ^{alt} collapsing edges of the bottom circl

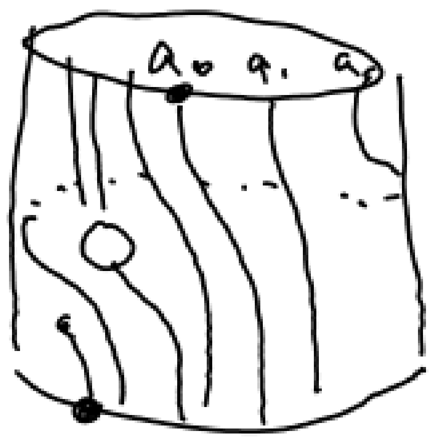
The 'action' $F : Cyl_* \otimes C^*(A, A) \to O(1, 1)$ is :

" sum of all meaningful interpretations as ops "

if $D \in C^*(A, A)$,



means $D(a_{j+k}, \dots, a_{j+k})\, a_0 \otimes a_1 \otimes a_2 \dots$



$\dots \sum (-1)^{\bullet}\, 1 \otimes a_{s+1} \otimes \cdots \otimes D(a_{\dots}, \dots) \otimes \cdots a_0 \otimes \cdots$

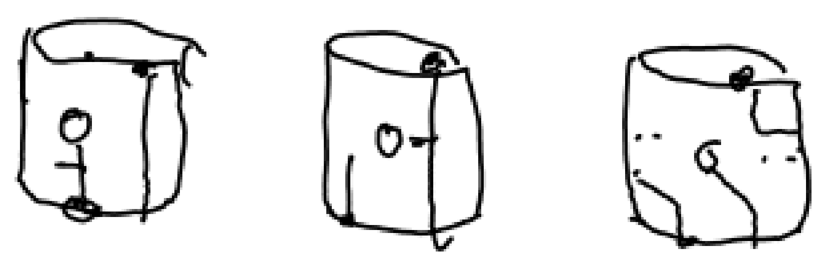
For $K \in Cyl_*$, $D \in C^*(A, A)$ put $F(K, D) = K_D$

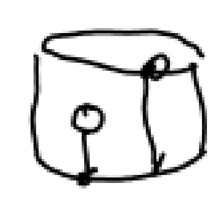
being a chain map means $dK \otimes D + K \otimes \delta D \mapsto (dK)_D + K_{\delta D}$

is eq to $[b, K_D]$

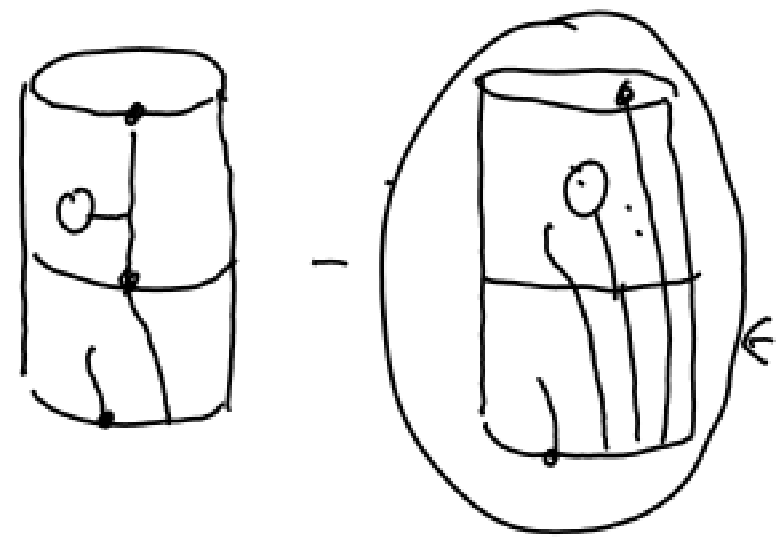
The CHF is $d"2" = B2 - 2B + L$

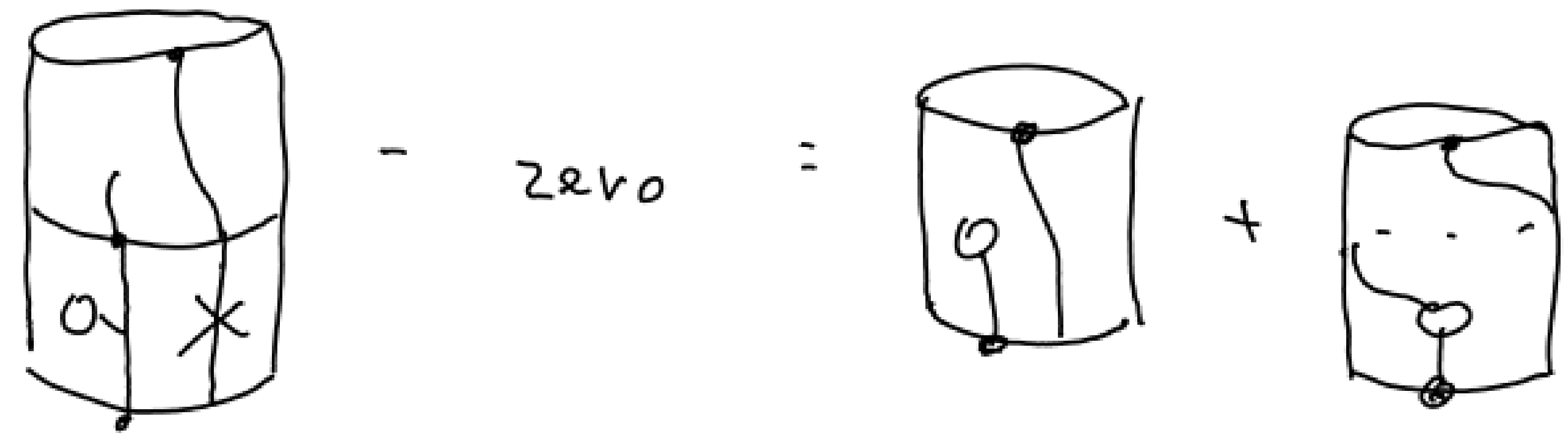
The left hand side =





suite

$B\iota =$  ← becomes Q (∵ acting on the reduced cplx)

$\iota B =$ 

(composition: either attach the end to bottom circle or puncture)